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Concrete - Representational - Abstract: An Instructional Approach in Teaching Integers

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Abstract

The study aimed to determine the influence of using Concrete- Representational -Abstract (CRA) as an instructional approach in teaching four operations of integers. The respondents of the study were the 47 Grade VII students of Nagsala High School (Domingo Ledesma Mapa High School) in Hda. Mapa, Sta. Cruz Viejo, Tanjay City. The quasi-experimental design was utilized. The researcher used a validated test instrument and employed the percent, weighted mean, and t-test methods in treating the data. The result of the pretest given to the two groups of students showed that the students did not meet the passing level. The study revealed that the utilization of CRA in the experimental group and the inductive approach in the control group developed the knowledge and skills of the students in the four operations of integers. The results further disclosed that the experimental group reached a satisfactory rating, while the control group reached a fairly satisfactory level only. Moreover, there was a significant difference in the posttest performances of the experimental and control groups in favor of the former group. Hence, using CRA as an instructional approach in teaching operations of integers was better than the inductive approach.

Keywords: CRA, Experimental, Traditional, Operations of Integers

Introduction

Teachers play a huge role in the development and achievement of students. They are responsible for molding every individual holistically. The teachers are also a very important element in the teaching-learning process considering that learners are diverse and thus have different learning styles, too. They bring the essential instruments for the student's achievement, specifically in challenging subjects like Mathematics, since they are influential as to how and what information is conveyed to students (Hughes, 2011).

Mathematics instruction is a discipline mainly related to abstract concepts. Effective mathematics instruction is an appropriate balance that is neither entirely 'teacher centered' nor 'student centered' (NMAP, cited in Hughes, 2011). However, according to Piaget (cited in Kontaş, 2016), students cannot understand and learn the abstract mathematical concepts just by way of pure words and symbols or simply through direct explanation and discussion.

Rubin et al. (2014) maintained that many students enter high school level with severe gaps in their basic concepts and skills in Mathematics, specifically performing integer operations—a necessary prerequisite skill to solve equations since learners are expected to expand their mathematical world in lessons like negative integers, (nonnegative), rational numbers and real numbers (Bruno & Martinón, cited in Whitacre, 2011).

However, Janvier and Vlassis, as cited in Bishop et al. (2011), added that integers and integer operations bring conceptual difficulties for students. Performing operations on integers involves the signs of the numbers and the signs of the required operation. Many students struggle with understanding integers and the accuracy of computing with integers (Harris, 2010) making them become confused when performing integer operations (Muñoz cited in Rubin, 2014). Consequently, different approaches are developed in order to address these needs, and one of these approaches is Concrete-Representational-Abstract (CRA). According to The Access (2009), the CRA sequence of instruction provides a graduated framework for students to create a meaningful connection, which starts with visual, tactile, and kinesthetic experiences to establish understanding. Students expand their understanding through pictorial representations of concrete objects and move to the abstract level of understanding. Indeed, any concept in Mathematics can be better understood when students connect these abstract ideas to concrete ones. The use of concrete materials in teaching and learning Mathematics, therefore, makes abstract concepts interesting and real (Rubin et al., 2014).

The researcher personally believes that the teacher has a significant role in the Mathematics achievement of the students. This conviction motivates the researcher to use the CRA instructional approach and determine its influence on teaching integers. The result of the study may serve as a useful aid for Mathematics teachers in using CRA as an intervention in Mathematics instruction and could be a tool to improve students' academic performance.

Literature Review

This section aims to cite literature and studies focusing on the use of Concrete-Representational-Abstract (CRA) as an instructional approach in teaching operations of integers to the Grade 7 students at Domingo Ledesma Mapa High School, Hda. Mapa, Sta. Cruz Viejo, Tanjay City, Negros Oriental, for the School Year 2018–2019.

Concrete - Representational - Abstract (CRA). CRA is a gradual systematic approach that has been found to be highly effective in teaching mathematical concepts. This approach primarily focuses on conceptual understanding and on students' demonstration of their mastery in the content (Flores, 2010). CRA involves three stages; each stage builds on what is taught in the previous stage. Therefore, concepts must be delivered or presented in an interconnected instructional sequence where each lesson relates to the previous lesson(s)**.**

CRA is a combination of effective components of both behaviorist (direct instruction) and constructivist (discovery-learning) practices (Sealander et al., cited in Garforth, 2014). It includes discovery-learning strategies involving representation to help students' transition between conceptual knowledge and procedural knowledge (Sealander et al., cited in Garforth, 2014). Steedly (2008) stressed that direct instruction refers to an instructional practice that carefully constructs interactions between students and the teacher. The author added that with this approach, the teacher clearly leads the teaching-learning process as CRA follows a defined instructional sequence. Moreover, Rupley (2009) maintained that the lying at heart of direct instruction are explicit explanations, modeling, demonstrating and guided instructions combined with student practice and feedback to teach thinking skills (Yeh, 2009).

According to Garforth (2014), CRA uses the common aspects found in direct instruction: a) demonstration**,** which refers to the way the teacher teaches, leads and exposes students to a step-by-step process (Star, 2008); b) modeling, where the teacher is doing and demonstrating the concept while involving the students in the process; c) guided practice, where the teacher helps and guides the students (Garforth, 2014); d) *i*ndependent practice, where students are given the opportunity to *practice* the concept by themselves and work towards mastery of the knowledge/skills presented to ensure that they succeed in the lessons before an assessment (Afflerbach, 2008); and e**)** feedback, which is defined by Shute (2008) as information communicated to the learners which guides them in the learning process and gives them the direction they need to reach the target or goal of the lesson, which is intended to modify their thinking or behavior and improve learning.

Impact of using CRA. Bender (2009) explained that CRA provides students with a structured way to learn mathematical concepts. In other words, CRA helps students to learn concepts first before learning rules. Thus, it enables students to build a better connection when moving through the levels of understanding from concrete to abstract, taught explicitly using a multi-sensory approach which makes learning accessible to all learners, including those with Mathematics learning disabilities.

CRA can be used for small groups or for the entire class. Thus, it helps students to learn with multiple intelligences. It caters to students who think in pictures and create a mental image to retain concepts (Visual- Spatial), students who possess Verbal-Linguistic intelligence, students who learn through hands-on activities (Bodily-Kinesthetic), and those who use logic to organize, classify and categorize information, and make connections and build relationships between ideas (Logical- Mathematical). CRA helps organize words in Mathematics problems in a way that makes sense.

Additionally, CRA teaches conceptual understanding by connecting concrete understanding to abstract mathematical process. Using CRA makes passive learners create meaningful connections. By linking learning experiences from concrete to representational to abstract levels of understanding, the teacher provides a graduated framework for students to make meaningful connections. In short, CRA blends conceptual and procedural understanding in a structured way.

Researchers claimed that utilization of CRA is effective to the learners' math fluency since it allows students to see and feel the attributes of the objects they are using (MathVIDS, 2012). The use of CRA is an effective instructional tool for students in learning various mathematical concepts. In fact, it also assists students in using concrete manipulatives, drawings, and then numbers in order to understand deeper the abstractness of Algebra (Stroizer, 2012).

Meanwhile, Steedly et al. (2008) proposed that CRA actually refers to a simple concept that has proven to be an effective method of teaching Mathematics to students, especially those students with disabilities. During the learning process, students can hold the objects in their hands and work with them concretely. This way, students are actually building a mental representation and image of reality being explored physically.In addition, Hauser (2009) explained that students who use concrete materials develop more precise and more comprehensive mental representations, often show more motivation and on-task behavior, understand mathematical ideas, and better apply these ideas to life situations. Carmichael et al. (2016) supports this notion stating that with CRA students become fluent in manipulating concrete materials and become able to explain the connection of the skills necessary for solving problems.

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Hughes (2011) also concluded that CRA is an effective instructional design because it explicitly teaches new information through a systematic and cohesive sequence of concepts. He added that teacher-and-students interaction allows the students to demonstrate perfect practice of the Mathematics constructs, which promotes mastery and fluency. Moreover, the study of Strickland and Maccini (2012) attested that the use of CRA with the integration of the concrete manipulatives does improve students' conceptual understanding and increases mathematical fluency. Kontaş (2016) also observed that using manipulatives (concrete stage) increases Mathematics achievement. Besides this, the students become more active, their motivation for learning grows higher, and they adopt a positive attitude towards Mathematics when manipulatives are employed in Mathematics classes (Enki, 2014; Gürbüz, 2007; Sowell, 1989; Yağcı, 2010).

Witzel, Riccomini and Schneider (2008) developed an acronym—CRAMATH—that teachers can use to assist them in creating their own CRA instructional sequence. CRAMATH outlines seven steps teachers can follow to create a mathematical unit:

1. **C**hoose the math topic to be taught.
2. **R**eview procedures to solve the problem.
3. **A**djust the steps to eliminate notation or calculation tricks.
4. **M**atch the abstract steps with an appropriate concrete manipulative.
5. **A**rrange concrete and representational lessons.
6. **T**each each concrete, representational and abstract lesson to student mastery.
7. **H**elp students generalize what they learn through word problems. (p. 273)

A research cited in Terry Anstrom(2012) revealed that "students who use concrete materials develop more precise and more comprehensive mental representations, often show more motivation and on-task behavior, understand mathematical ideas, and better apply these ideas to life situations." Research also shows that using the CRA approach works effectively especially for students who have a learning disability in Mathematics and that students are more apt to gain and retain an understanding of mathematical concepts when they are taught using CRA.

Toong (2015) in her study entitled "The Extent of Use of Concrete-Representational-Abstract (CRA) Model in Mathematics" revealed that the extent of teachers' use of CRA model was generally "high" in all areas in Mathematics. It was suggested that teachers should devise a strategy or approach appropriate to students to help them improve their performance in Mathematics and should conduct an experimental study on the use of CRA to identify its effect.

Models for teaching integers

Algebra tiles. Using Algebra Tiles gives students a visual, hands-on way of exploring patterns at the introductory stage of learning a new concept. This hands-on approach, which uses a narrative style in explaining situations, allows students to state the rules of integers and algebra from their own experience as they communicate with each other. Once students have seen the patterns and stated the rules, the tiles can be put aside. Students then extend rules to more complicated examples using only the symbolic form (MATHGains).

The charged particles model. This utilizes positively and negatively charged particles to perform addition and subtraction operations.

The stack or row model. This model uses colored linking cubes and graph paper. Students use the graph paper and colored pencils for recording problems and results. Students create stacks or rows of numbers with the colored linking cubes and combine/compare them. If the numbers have the same signs, then the cubes are also of the same color. The cubes are then combined to make a stack or row. However, if the numbers have unlike signs (colors), the stacks of different colors are compared, the tallest stack wins. The result is the amount of difference between the stacks and is easy to see and understand.

The hot air balloon model. Sandbags (negative integers) and Hot Air bags (positive integers) can be used to illustrate operations with integers. Bags can be put on (added to) or taken off (subtracted from) the balloon.

The number line model. This model requires counting the directional distance from "a to b" and "b to a." Number lines allow students to not only compare integers, but also begin to view positive and negative numbers as opposite (TalkingWithMyHands, 2015).

Postman stories. This is a story about a postman who only brings financial mail. In other words, it is simply a story that involves money.

Mnemonic. This entails using and singing the melody of "Row, row, row, your boat." The students are given the following words: "Same Sign, Add and Keep, Different Sign, Subtract. Take the sign of the higher number then it'll be exact!"The teacher may sing it a few times and dance around a little with the students.

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Flores (2010) further validated her previous CRA research (Flores, 2009) to include subtraction with regrouping using more complex computation. Flores' study investigated the effectiveness of the CRA sequence in teaching subtraction with regrouping to the tens and 45 hundreds place. The results of the study provided evidence that there is a functional relation between CRA and the students' performance in subtraction with regrouping to the tens and hundreds place. It further validated that CRA is indeed an effective instruction sequence for teaching students with learning disabilities and those who have difficulty in subtraction with regrouping to the tens and hundreds place. Meanwhile, Mercer and Miller, as cited in Flores (2010), taught basic addition, subtraction, multiplication, and division to students with high-incidence disabilities using either the CRA sequence or the traditional curriculum. Mercer and Miller found that students performed significantly better when instruction involved the CRA sequence and the mnemonic strategy.

In the study of Sengul and Dereli (2013) entitled "The Effect of Learning Integers Using Cartoons on 7th Grade Students' Attitude to Mathematics," data revealed that, as compared to the conventional teaching method, teaching using cartoons significantly increases students' perceived level of Mathematics achievement, the perceived benefits of mathematics, and the amount of attention students pay to their Mathematics subject.

The study of Dunne et al. (2017-2018) entitled "Lesson-Research Proposal for 1st Tears-Integers" revealed that a lot of factors cause students to not fully understand concepts. One of these factors is that students acquire minimal learning if during the delivery of instruction, the teacher does not always give necessary time that would allow them to manipulate, explore and come up with their own way of answering and demonstrating their understanding. This finding is also supported by the study of Reisa (2010) entitled "Computer-supported Mathematics with GeoGebra," which found that most of the students fail to understand concepts using conventional teaching because it only appeals to the auditory sense. Furthermore, Hughes (2011) in his study "The effects of concrete-representational-abstract sequenced instruction on struggling learners acquisition, retention, and self-efficacy of fractions" revealed that students who were taught using the conventional instruction received less modeling and guided practice since conventional teaching requires students to create meaning and potentially learn through trial-and-error instead of mastery by accurate practice.

Another study entitled "Activity-Based Teaching of Integer Concepts and its Operations" by Rubin et al. (2014) showed solid evidence that when students are engaged in activity-based teaching, a positive learning experience takes place, and students achieve a better understanding of the concept. Thus, activity-based teaching introduced by the teacher can improve students' conceptual understanding, perception and procedural skills in integers. Furthermore, the use of activity-based teaching shapes the way students think and build connections between concepts, thus fostering an increase in student retention. Hence, students eventually can extend their conceptual understanding of addition, subtraction, multiplication and division to positive and negative numbers.

Other Instructional Materials

The study of Sapuan (2013) entitled "Utilization of Games in Introduction of Mathematical Concepts in Relation to the Pupils' Academic Performance" revealed an "advance" mark in students' performance in all the Mathematics games. It was suggested that teachers should encourage students to engage more in the activities to have more chances of getting a good grasp on the concept being taught.

Similarly, the study of Panzo (2012) entitled "Instructional Materials and Activities in Mathematics: Their Impact on High School Respondents' Academic Performance" revealed that the respondents who actively and almost always participated in different Math activities showed better academic performance. That is why teachers are encouraged to use sufficient instructional materials in teaching Mathematics.

Also, the study of Futalan, as cited in Toong (2015) entitled "Solutions Manual in Selected Topics in Differential Calculus: Its Effect on Students' Achievement," showed that the mean achievement of the whole group of sampled students in lessons taught with the solutions manual was significantly higher than the mean achievement in lessons taught without the solutions manual. Thus, her findings simply indicate that instructional materials contribute to students' performance.

Student's difficulty in learning integers

According to Henley, as mentioned in the study of Whitacre et al. (2011), subtraction of integers is a sense in which a subtrahend can be regarded as a subtractive number which means that the subtraction operation is included into the quality of the number. Henley's study provided this example: instead of interpreting the expression 9 – 5 as subtract 5 from 9, students might instead think in terms of 9 and "subtract 5" being combined additively. Under this interpretation, "subtract 5" becomes an entity that one can think about, even in the absence of any particular minuend.

On the other hand, Rubin et al. (2014) revealed that students get confused when asked to perform operations on integers, which involves the signs of the numbers and the signs of the required operation. What makes it especially difficult is when students are taught to follow rules and procedures in a very abstract manner without going through models for better conceptual understanding. This claim agrees with the finding of Ural (2016) that students have difficulties in ordering integers and performing their operations. Similarly, Janvier, Hativa, and Cohen found that students struggle with understanding the concept of negative numbers and the conflict between the two different meanings of the sign and the signed numbers (cited in Ural, 2016).

Correspondingly, Harris (2010) affirmed that students' lack of understanding of the concept and rules of subtraction of integers is the most difficult and challenging operation for students. The author further asserted that the use of parentheses for either subtraction or addition with a negative number is more confusing to them. In other words, students find it difficult to understand the rules in operations like subtracting a negative integer from a positive integer. The same is true with adding a positive integer to the opposite of a negative integer. Students have difficulty understanding that the rules of signs in multiplication and division are the same and the idea of subtracting integers may result in a positive integer.

In addition, Bishop, et. al (2011) posited that students most likely think that negative numbers are greater since it comes before positive ones (e.g. -7 is more than 3). Likewise, Freudental et al., as quoted by Nool (2012), found out that students experienced extreme difficulties in conceptualizing and performing operations with negative numbers in the pre-algebraic and algebraic scope. Betaño and Gamido also noted that college freshmen had difficulties in performing addition and subtraction of signed numbers.

According to Nool (2012), future elementary teachers had much difficulty in adding integers. Their difficulty in this operation was quite alarming considering that they had studied this topic since first year high school. This difficulty could be attributed to their inability to comprehend the rules applied to this operation. This is where physical representations come in to see connections between the abstract and concrete enabling learners to understand and develop mental images of this process.

Materials and Methods

This study made use of the quasi-experimental design. With this design, only the instructional approaches were randomly assigned to every class section through tossing a coin. There were no random assignments of respondents to every section. A pretest was given before the treatment (utilization of two approaches) in teaching the topic and a posttest was administered after the treatment.

The setting of this study was at Domingo Ledesma Mapa High School where the researcher has been teaching Mathematics for four years. This secondary institution is a medium secondary school in the division of Tanjay City. Situated in Hda. Mapa, Sta. Cruz Viejo, Tanjay City, Negros Oriental, this secondary school is 7 kilometers away from the city. It was founded on August 4, 2008.

Currently, the school has a population of 321 students (260 students in Junior High School and 61 students in Senior High School) and 14 nationally-funded teachers. The school has a sufficient water and electricity supply.

The respondents of this study were the Grade 7 students of Domingo Ledesma Mapa High School for the school year 2018-2019. There were two groups that were randomly assigned as the experimental group and the control group. The number of students per section were:

Section	**No. of Students**
Grade 7 - A (experimental group)	25
Grade 7 - B (control group)	22
Total	**47**

To determine the performance of the students in the four operations of integers, the researcher utilized the validated test instrument of Estoconing (2015). It was a 32-item test. A dry run was conducted among Grade 10 high school students not covered in this study. The test was administered to the respondents before discussing the lessons using different approaches, and after discussing the lessons using the CRA and Inductive approaches. The Test-Retest method was also employed to determine the coefficient reliability, which is equivalent to 0.820. This figure signifies that the items are reliable and acceptable since it was greater than 0.070.

There were two classes that consisted of Grade 7 students. The same topics were taught in each class. Two different approaches were utilized but were randomly assigned to every class through a coin toss. As a result, Grade 7-A was assigned as the experimental group to undergo the CRA approach, and Grade 7-B was assigned as the control group to undergo the Inductive approach. A pretest was conducted before the discussion of the topics using the approach assigned.

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The researcher, as the teacher, utilized the CRA instructional approach through the giving of concrete materials, specifically apples and cut-out pictures of apples. This was done to provide students with the opportunity to work with these tangible materials using their senses. Representations of these materials were also provided to help students arrive with symbols and numbers in solving the four operations of integers.

The utilization of Inductive approach was done by presenting specific problems through plain presentations, videos, questions-answers, provision of examples, showing and practice sessions, where students were asked to give their ideas. Afterwards, the teacher proceeded to discussing the general rules in solving the four operations and their application.

After the discussion and the utilization of the two approaches, the students answered the posttest. This determined which particular approach had greater influence on the students' performance in Mathematics.

Results

This section provides information about the data gathered from the scores of the Grade 7 students of Domingo Ledesma Mapa High School. The data are presented in tabular and textual forms, analyzed, and interpreted to answer the problems presented earlier in this study.

Table 1: *Pretest Performances of the Experimental and Control Groups of Students*

Group	Mean (%)	Verbal Description	Standard Deviation
Experimental Group (n = 25)	71.88	Did Not Meet Expectations	4.28
Control Group (n =22)	69.59	Did Not Meet Expectations	3.95

Legend:	**Rating**	**Verbal Description**
	90% - 100%	Outstanding
	85% - 89%	Very Satisfactory
	80% - 84%	Satisfactory
	75% - 79%	Fairly Satisfactory

Table 2 *Posttest Performances of the Experimental and Control Groups of Students*

Group	Mean (%)	Verbal Description	Standard Deviation
Experimental Group (n = 25)	82.16	Satisfactory	10.87
Control Group (n =22)	74.73	Fairly Satisfactory	11.03

Legend:	**Rating**	**Verbal Description**
	90% - 100%	Outstanding
	85% - 89%	Very Satisfactory
	80% - 84%	Satisfactory
	75% - 79%	Fairly Satisfactory
	Below 75%	Did Not Meet Expectations

Table 3

Analysis Table on the Difference between the Pretest and Posttest Performances of the Students

Group	Pretest (%)	Posttest (%)	Difference (%)	t-value	p-value	Decision	Remark
Experimental Group (n = 25)	71.88	82.16	10.28	5.319	0.000	Reject H_{o1}	Significant
Control Group (n =22)	69.59	74.73	5.14	2.768	0.005	Reject H_{o1}	Significant

Level of significance = 0.05

Table 4
Analysis Table on the Difference between the Posttest Performances of the Experimental and Control Groups

Group	Posttest (%)	Difference	t-value	p-value	Decision	Remark
Experimental Group (n = 25)	82.16	7.43	2.404	0.020	Reject H_{o2}	Significant
Control Group (n =22)	74.73					

Level of significance = 0.05

Discussion

The data in Table 1 showed that the pretest performances of the students before teaching them using the two different approaches are 71.88% for the experimental group and 69.59% for the control group. This shows that the two groups of students did not meet the expectations for the four operations of integers. This means that these groups of students still struggle with understanding the process involved in the four basic operations of integers (DepEd Order No. 8, s. 2015). Based on the outputs, students had difficulty following the rules in adding and subtracting integers.

This strengthens the finding of Janvier, Hativa, and Cohen, as cited in Ural (2016), that students have difficulties understanding the concept of negative numbers and the conflict between the two different meanings of the sign and the signed numbers. This is also in line with the result of Ural (2016), which revealed that the students struggled with ordering the integers and performing their operations.

Similarly, Rubin et al. (2014) found that students get confused when asked to perform integer operations, which involve signs of the numbers and the signs of the required operation. This aspect becomes especially difficult for students if they are taught to follow rules and procedures in a very abstract manner without going through models for better conceptual understanding.

In addition, Freudental et al., as mentioned by Nool (2012), found out that students experienced extreme difficulties in conceptualizing and performing operations with negative numbers in the pre-algebraic and algebraic scope. Moreover, Bishop et al. (2011) asserted that students most likely think that negative numbers are greater since they come before the positive ones.

The data in Table 2 reveal the posttest performances of the students in the four operations of integers after teaching the topic using CRA approach and Inductive approach. The data reflect that the experimental group is in the satisfactory level (82.16%). This means that students at this level have developed the fundamental knowledge and skills and core understandings of the topic and with little guidance from the teacher and/or with some assistance from peers and can transfer these understanding through authentic performance tasks (DepEd Order No. 8, s. 2015). This also means that the students in this level have acquired moderate and sufficient concepts of the four operations of integers needed to transfer learning towards another task. It has to be noted that the teacher in the experimental group utilized CRA in imparting the concepts of the four basic operations of integers.

Researchers claimed that utilization of CRA is effective for learners' math fluency since it allows the students to see and feel the attributes of the objects they are using (MathVIDS, 2012). By holding the objects in their hands and working with them concretely, students are building a mental representation and image of reality being explored physically (Steedly et al., 2008). Moreover, the students can explain the connection of the skills (Carmichael, et.al, 2016), thus improving their understanding of the four basic operations of integers(Ural, 2016).

This finding is supported by Hughes (2011), who concluded that CRA is an effective instructional design since it explicitly teaches new information through a systematic and cohesive sequence of concepts. According to Hughes, the teacher-and-students interaction allows the students to demonstrate perfect practice of the Mathematics constructs, which promotes mastery and fluency. In addition, Hauser (2009) postulated that students who use

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concrete materials develop more precise and more comprehensive mental representations, often show more motivation and on-task behavior, understand mathematical ideas, and better apply these ideas to life situations.

On the other hand, the performance of the students who were introduced to the inductive approach of teaching integers in the control group is at a fairly satisfactory level (74.73%). The results indicate that the students at this level possess the minimum knowledge and skills and core understandings of the topic and that they need help throughout the performance of an authentic task (DepEd Order No. 8, s. 2015). These students also require assistance, such as remedial classes and tutorials from their teachers or peers in order to gain more knowledge of the four operations of integers.

Several factors cause students to not fully understand the concepts. According to Dunne et al. (2017-2018), students acquire minimal learning if, during the delivery of instruction, the teacher does not always give the students the necessary time that would allow them to manipulate, explore, and come up with their way of answering and demonstrating their understanding.

Likewise, Reisa (2010) discovered that most students fail to understand concepts with conventional teaching because it only appeals to the auditory sense and students may get bored and easily distracted. This is affirmed by Kontas (2016) who claimed that students who experience manipulating concrete objects and concrete materials learn better the abstract mathematical concepts.

The data in Table 3 indicate that there is a difference of 10.28% in the pretest and posttest performances of the students in the experimental group. To test the data statistically, a t-test for dependent data was applied. The data then revealed that the p-value (0.000) is less than the level of significance (0.05). This finding is enough evidence to reject the null hypothesis. This means that a significant difference exists between the pretest and posttest performances of the students in favor of the latter performance. Further, this implies that the utilization of CRA in the experimental group enables the students to grasp the concepts of the four basic operations of integers.

Correspondingly, Mercer and Miller, as cited in Flores (2010), affirmed that students perform significantly better when instruction involves the CRA sequence, and they outperform their peers who receive traditional instruction at the abstract level.

As also reflected in the Table, a difference of 5.14% is apparent between the pretest and posttest performances of the students in the control group. Applying a t-test for dependent data, it is shown that the p-value (0.005) is less than the level of significance (0.05). This figure is sufficient evidence to show that a significant difference exists between the pretest and posttest performances of the students in favor of the latter performance. This signifies that students also learn using the traditional method of teaching the basic operations of integers, although the posttest rating (74.73%) is just barely passing.

Indeed, Hughes (2011) revealed that students taught using the conventional instruction receive less modeling and guided practice since the traditional method requires students to create meaning and potentially learn through trial-and-error instead of mastery by accurate practice. The author also asserted that students store correct algorithms in their short-term memory but never move the information to their long-term memory and therefore are not able to retrieve necessary information at a later date.

Furthermore, Rubin et al. (2014) emphasized that the true meaning of the ability to learn is not just to memorize the rules of a particular task, and that it is not enough to have discussions of the rules in operations of integers. He concluded that exposing students to make Mathematics come alive and make it more concrete and tangible for them is an effective way to improve students' procedural skills and conceptual understanding of integer operations.

The data in Table 4 show a difference of 7.43% between the posttest performances of the students. To test the data statistically, a t-test for independent data was applied. Results revealed that the p-value (0.020) is less than the level of significance (0.05), which warrants rejection of the null hypothesis. This means that there is a significant difference between the posttest performances of the two groups of students in favor of the experimental group. This finding implies that the use of CRA enables the students to learn better the concepts of the basic operations of integers.
As attested by Strickland and Maccini (2013), the use of CRA is effective since the integration of Concrete-Representational-Abstract in teaching improves students' conceptual understanding and increases their Mathematics fluency. This is further affirmed by Stroizer (2012), who postulated that the CRA sequence is an effective instructional sequence for many mathematical concepts.

Conclusion

Based on the findings cited, the pretest performances of both groups of students did not meet expectation. When it comes to the approaches used having the same topic, the posttest performance of students subjected to CRA approach is at a satisfactory level while those students subjected to the inductive approach is in the fairly satisfactory level. Thus, there is a significant difference between the pretest and posttest performances of the students after utilizing two approaches in favor of the latter performances. The increase in their performances is attributed to the teacher's utilization of CRA and the inductive approach and to any other intervening factors of the study. In addition, there is a significant difference between the posttest performances of the students. Therefore, CRA as an instructional approach is better than the inductive approach in teaching the four operations of integers. In general, utilizing CRA is an effective approach and can be an intervention in improving the Grade 7 students' performance in the four operations of integers.

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